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Objective probabilities about future climate are a matter of opinion

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Abstract In this paper, the unfeasibility of producing "objective" probabilistic climate change scenarios is discussed. Realizing that the knowledge of "true" probabilities of the different scenarios and temperature changes is unachievable, the objective must be to find the probabilities that are the most consistent with what our state of knowledge and expert judgment are. Therefore, subjective information plays, and should play, a crucial role. A new methodology, based on the Principle of Maximum Entropy, is proposed for constructing probabilistic climate change scenarios when only partial information is available. The objective is to produce relevant information for decision-making according to different agents' judgment and subjective beliefs. These estimates have desirable properties such as: they are the least biased estimate possible on the available information; maximize the uncertainty (entropy) subject to the partial information that is given; The maximum entropy distribution assigns a positive probability to every event that is not excluded by the given information; no possibility is ignored. The probabilities obtained in this manner are the best predictions possible with the state of knowledge and subjective information that is available. This methodology allows distinguishing between reckless and cautious positions regarding the climate change threat.

1 Introduction

Efficient use of economic resources to cope with global warming in terms of adaptation, mitigation and impacts (remediation and avoidance) depends on the

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amount, quality and interpretation of the information, and of the known uncertainties. Decision-making and risk assessment critically depend on how uncertainty is managed.

A great deal of effort in climate change has been devoted to reduce uncertainty (of different sources) in order to increase the usefulness of future climate scenarios for the assessments of the potential impacts on human and natural systems (IPCC 2001a). In recent years, great advances have been developed in making "the unknown known" (Schneider 2003): an increasing understanding of the climate system has permitted to improve the models' complexity and ability to mimic it, providing more confidence in climate scenarios, higher resolution in time and space, as well as the possibility of constructing probabilistic climate change scenarios using different storylines and models. As noted by Schneider (2001), the decision of the IPCC, and in particular of the SRES (Special Report on Emissions Scenarios. IPCC 2000) authors, of not assigning probabilities of occurrence to any emission scenario in order to not reveal any preference or policy recommendation, leaves decision-makers the dirty job of assigning arbitrary probabilities to temperature (and precipitation) changes in order to make this information useful for assessing the possible impacts. The fact that the IPCC qualifies every emission scenario as equally sound and that it states that any particular scenario is "no more or less likely than any other scenarios" has lead many to assume that it is most reasonable to assign a uniform distribution to the set of emission scenarios.[1]

Schneider (2001) showed that giving the same probability to each emission scenario leads to bell-shaped histograms for global temperature for year 2100. He argues that, even if emission scenarios were considered to be equally probable, the resulting temperature scenarios would not be so.[2] Evidently, this information would be very valuable for decision-making. Grübler and Nakicenovic (2001) questioned the appropriateness and feasibility of assigning (subjective or objective) probabilities to future scenarios and state that Schneider's position might be dangerous because it could lead to a dismissal of uncertainty. First of all, it is important to clarify that there is no "special uncertainty" in emission scenarios (and therefore in climate change scenarios) that prevents the use of (subjective) probabilities.[3] This is a fake distinction of "climate change" uncertainty. In addition, although they provide some valuable arguments regarding the unsuitability of using relative

[1] Although this could be perhaps the most "honest" way of doing so, it is also debatable: there is no evident symmetry in emission scenarios that makes them clearly equally probable. Low emission scenarios imply political, economic and social efforts that, as can be learned form Kyoto, are not easily attained, while "business as usual" or "higher emission" type scenarios do not. It seems that the climate change threat, under the prevailing veil of skepticism and uncertainty, has not yet balanced the odds enough to fully support a symmetry argument. Therefore, for many people, assigning the same probability to each emission scenario could also be considered as arbitrary as any other probability assignment.

[2] This argument assumes that probabilities and frequency ratios are the same. As will be discussed later, this assumption has several inconvenients and indeed can dismiss uncertainty in an unjustifiable manner.

[3] Here we make the distinction between uncertainty and ignorance. Clearly it can be unfeasible to assign probabilities to outcomes which we are ignorant about their possible occurrence. Here we are dealing only with the known uncertainty reflected by SRES and the different climate models.

frequencies to infer probabilities of future scenarios,[4] we believe that they missed the main issue brought forward by Schneider and the fact that there is an equal (or greater) 'danger': freezing up the decision-makers' capacity to take any action given the high level of uncertainty surrounding the climate change threat. Illustrative examples of this situation occur when presenting precipitation scenarios of the type "*n* scenarios say that precipitation will decrease about a 30% but *m* scenarios say that precipitation will increase about a 30%" to stakeholders and decision-makers. Two basic interpretations are commonly drawn: (1) scientists have no idea of what is going to happen, and therefore no information can be extracted from climate change science that is useful for decision making. Many stakeholders and decision-makers typically conclude that the best thing to do is nothing or to postpone decision taking until further information is available. (2) There is a $m/(n+m)$ chances of an increase in precipitation and an $n/(n+m)$ of a decrease. This is clearly an abuse of relative frequencies as probabilities. Both interpretations can be misleading and science is failing to communicate to social agents one of main the basic row materials for decision-making and risk assessment: the probability of occurrence of the different outcomes. Without such estimates, risk assessment methodologies cannot be applied, nor decision theory tools such as expected utility, for example. A great deal of the social relevance of climate change science, if not all, is in function of its applicability for decision-making.

As stated in Schneider (2003) "two options are appropriate in the face of uncertainty: (1) reduce uncertainty through data collection, research, modeling, simulation, etc. and (2) manage or integrate uncertainty into the decision-making or policy-making process". Probabilities are required for this latter option and for taking advantage of recent advances in our understanding and modeling of the climate system. However, undue reduction of uncertainty—for example, excluding or assigning arbitrary low or high probabilities to certain models or scenarios further than the information available allows to—can generate important biases on risk assessment and therefore critically limit policy efficacy and economic efficiency.

At this point, it is convenient to recall that frequencies and probabilities are closely related but are different concepts. From a frequentist perspective, the "objective" probability of an event, which can be empirically measurable by observation in a random experiment, can be well approximated by frequency ratios as the number of repetitions approaches infinitum. The predictions obtained by assigning a probability distribution in this manner are in principle verifiable and if probabilities were correctly assigned they must adequately represent the variations of the random variable of interest (see Jaynes 1957). The assignment of probabilities via frequency ratios to climate change scenarios has, therefore, several inconvenients: (1) the event under study is not observable and cannot be empirically measurable (see Kinzig et al. 2003); (2) the range of outcomes is not produced by a *random experiment* which implies a number of repetitions under the *same* conditions and the same

[4]Nevertheless, they assert a rather artificial differentiation of natural sciences and social sciences probabilities, implying that natural sciences' are frequentist while social sciences' are subjective. This is not true and it might be convenient to recall, for example, the influential work of Jaynes in thermodynamics and statistical mechanics. On the other hand, for example, a large part of the published work in econometrics is based on the frequentist approach. It's also important to underline the arguments of Schneider (2002) regarding that the 'path dependence' is part of both natural and social systems.

underlying data generating process: the use of different emissions scenarios (and/or models) prevent this; (3) we have only a limited number of outcomes from which we must infer frequency ratios. It is convenient to recall that when using Monte Carlo methods to infer probability distributions, in most cases, several thousand repetitions are needed. Furthermore, if we take a close look at emission and temperature figures shown in the IPCC Third Assessment Report (IPCC 2001a), scenarios tend to cluster and produce "gaps" for certain values. These gaps imply zero probability for some intervals and are clearly not the product of any physical reason but a *sampling* problem. This problem is not trivial and can generate important bias in the probability distribution that is calculated; (4) it cannot be verified how well the probability distribution fits reality; we have no observations for, say, 2100; (5) climate change scenarios might tend to cluster over a particular value because they rely on the same information and because the models used might share similar modeling strategies (climate models can hardly be considered independent), and not necessarily because this particular value has greater probability than any other. Reading the same news on two different newspapers that rely on the same information source does not make it more believable (Allen 2003).

The broad range of outcomes produced by the different climate change scenarios only reflects our *state of knowledge* which we need to optimally process in order to attain "best response" type of decision-making. The complexity of the systems (as well as their relationships and feedbacks) in study insure that the information available for decision-making will always be partial and that "objective" scientific information will have to be complemented with "subjective" (expert) judgment.

Therefore, it is important to realize that we have to resign to ambitious questions such as "how likely is it that the world will get 6°C hotter by 2100?" (Schneider 2001). That is not known and will remain so until we reach that year (of course our uncertainty will be smaller as we approach this date). Instead, we have to ask a much modest question: given our state of knowledge, how strongly *we believe* the world will get 6°C hotter by 2100? Evidently, there is no unique answer to this question.

Since it is impossible to know the "true" probabilities of the different scenarios and temperature changes, the objective must be to find the probabilities that are the most consistent with what our state of knowledge and expert judgment are. If this is the case, optimal decision-making can be achieved: no matter what the actual outcome is, and therefore, if the decision was right or wrong, it is still the optimal response given the information (objective and subjective) available. In this sense, the validity and usefulness of probabilistic climate change scenarios constructed in this manner do not depend upon the true future climate values.

In this paper, we present the Maximum Entropy Principle as a useful tool for constructing probabilistic climate change scenarios that are the least biased estimates possible, consistent with the information at hand (including expert or decision-maker judgment) and that maximize what is not known. This warrants that no additional, unintended assumptions are made and assigns a positive probability to every outcome that is not absolutely excluded by the available information. Entropy provides a measure of the uncertainty associated to a particular probability distribution and therefore, how much information we are assuming to have when we assign it to an ensemble (set) of climate change scenarios. The Maximum Entropy Principle is presented as a simple and robust way of making use of both objective and subjective information, providing a systematic, unbiased and optimal approach for assisting decision-making under uncertainty.

2 Entropy and the maximum entropy principle

2.1 Information theory entropy

Shannon (1948) introduced the "information theory" entropy which provides a measure of the uncertainty associated to a given probability distribution. This measure can be expressed as

$$H(X) = -K \sum_{i=1}^{n} p_i \log(p_i)$$

Where X is a discrete random variable; p_i is the probability of state i and K (a positive constant) is used to modify the unit of measure. K = 1 and the natural logarithm will be used for the calculations presented in this paper.

It has been shown that information entropy is the unique measure of uncertainty of a probability distribution that satisfies the following properties (Shannon 1948; Jaynes 1957):

1. H is a continuous function of the p_i
2. If all p_i are equal, the quantity $A(n) = H(1/n, \ldots, 1/n)$ is a monotonic increasing function of n. As a system increases the number of possible states, and all states have the same probability of occurrence, the uncertainty about what the outcome will be is necessarily larger. If we trade a six-sided fair die for a ten-sided fair one, we would be more uncertain about the value of the outcome in the next toss.
3. The composition law. If a choice is broken down in two successive choices, the original H should be a weighted sum of the individual values of H.

Other properties of information entropy are: If all events are equally likely to occur then uncertainty reaches its maximum (uniform distribution); H $\geq$ 0 and is strictly equal to zero when for some i, $p_i = 1$ and for every $j \neq i$, $p_j = 0$. That is, i will occur with no uncertainty; $-\log(p_i)$ is referred as the surprise (*surprisal*) associated to the outcome i. If p_i is small, the surprise of learning that i was the actual outcome would be large, and if p_i is large the surprise that i occurred would be small. Therefore, H is the expected value of the surprise and provides a measure of the surprise of learning the value of X. $-\log(p_i)$ is also interpreted as the information we gain when we know that the event i occurred. Thus, the greater H, the more informative (on average) a measurement of X is.

These interpretations of entropy are useful for illustrating how different stakeholders' beliefs might (or might not) be consistent with different probability assignments. Consider, for example, assigning equally probabilities to every outcomes. Do we really believe that the surprise/information of learning that global temperature by year 2100 is 1.4°C or 5.8°C is the same? Are the actions taken by governments up to this date consistent with this probability assignment? As will be shown later, with this probability assignment the probabilities of surpassing some critical thresholds that have been defined as "dangerous" climate change are quite large.

2.2 Maximum entropy principle

Jaynes (1957, 1962) proposed the Maximum Entropy Principle using Shannon's entropy measure as a way for setting up probability distributions on the basis of partial knowledge. He showed that the maximum entropy estimate is the least biased estimate possible on the information at hand. This estimate maximizes the uncertainty (entropy) subject to the partial information that is given. That is, it produces the closest probability assignment to a uniform distribution consistent with what is known. The maximum entropy distribution assigns a positive probability to every event that is not excluded by the given information; no possibility is ignored (see Jaynes 1957).

In the following paragraphs the Jaynes (1957, 1962) Maximum Entropy formalism is presented.

Suppose the quantity x can take the values $(x_1, x_2, ..., x_n)$ where n can be finite or infinite and that the information expressed in terms of average values or moment constraints $\{f_1(x), f_2(x), ..., f_m(x)\}$ is given, and where m < n. The objective is to determine the probabilities for each possible value of x consistent with the information at hand. Using the Maximum Entropy formalism, the problem can be expressed as:

$$\max_p H\left(X\right) = -K\sum_{i=1}^{n} p_i \log\left(p_i\right) \tag{1}$$

subject to

$$\sum_{i=1}^{n} p_i f_k\left(x_i\right) = y_k,\ \ k = 1, 2, \ldots, m\ \ \left(\text{moment-consistency constraints}\right) \tag{2}$$

$$\sum_{i=1}^{n} p_i = 1\ \ (\text{normalization condition})\ \ \text{where}\ p_i \geq 0 \tag{3}$$

Note that the problem is ill-posed or underdetermined because there are n probabilities to be recovered but only $m + 1 < n$ data points, consisting of the m moment-consistency constraints and that the sum of the n probabilities must equal 1 (Golan et al. 1996). A Lagrangian function can be used to recover the probabilities that are consistent with the constraints (2) and (3):

$$L = -\sum_{i=1}^{n} p_i \ln\left(p_i\right) + \sum_{k=1}^{m} \lambda_k \left[y_k - \sum_{i=1}^{n} p_i f_k\left(x_i\right)\right] + \mu\left[1 - \sum_{i=1}^{n} p_i\right]$$

with the first order conditions

$$\frac{\partial L}{\partial p_i} = -\ln\left(\hat{p}_i\right) - 1 - \sum_{k=1}^{m} \hat{\lambda}_k f_k\left(x_i\right) - \hat{\mu} = 0$$

$$\frac{\partial L}{\partial \lambda_k} = y_k - \sum_{i=1}^{n} \hat{p}_i f_k\left(x_i\right) = 0$$

$$\frac{\partial L}{\partial \mu} = 1 - \sum_{i=1}^{n} \hat{p}_i = 0$$

The formal solution to this system of $n + m + 1$ equations and parameters is

$$\hat{p}_i = \frac{1}{\Omega\left(\hat{\lambda}_1, \hat{\lambda}_2, ..., \lambda_m\right)} \exp\left[-\sum_{k=1}^{m} \hat{\lambda}_k f_k(x_i)\right]$$

where

$$\Omega\left(\hat{\lambda}\right) = \sum_{i=1}^{n} \exp\left[-\sum_{k=1}^{m} \hat{\lambda}_k f_k(x_i)\right]$$

is a normalization factor called the partition function.

The maximum entropy distribution does not have a closed-form solution and therefore numerical optimization techniques are required for calculating the probabilities.[5]

Of the infinite number of probability distributions that are consistent with the given information (that satisfy the constraints), the maximum entropy distribution is the least biased estimate: any other probability assignment would imply assumptions not warranted by the available information. It allows to reason at best about the probability of the n possible outcomes (not their relative frequencies) and to make the best predictions consistent with the given information. This is not to say that predictions are correct, but that in order to improve them more information is needed (Jaynes 1957, 1962, 2003).

Although the Maximum Entropy Principle is not intended for estimating relative frequencies,[6] the probability that it assigns to the event x_i is equal to an estimate of the relative frequency of this outcome in an infinitely large number of trials: the maximum entropy distribution is the same as the frequency distribution that can be realized in the greatest number of ways consistent with the given information (see Jaynes 1962; Golan et al. 1996).

3 Risk assessment and the construction of probabilistic climate change scenarios

3.1 Choosing the state of knowledge

It is important to notice that it is also impossible to know the true range of possible changes in climate variables for, say, 2100. The current knowledge and the intrinsic epistemic uncertainty in climate (including surprises and feedbacks), socioeconomic and emissions processes and modeling, make this unattainable. Then, the objective must be to choose a state of knowledge that is physically plausible and that reflects the range of uncertainty available in the current literature.

In this section, some of the possible effects of choosing different ranges (states of knowledge) are shown. For illustrating purposes, consider the following examples.

[5] Nevertheless, this optimization problem can be solved using simple tools such as the Excel solver function.

[6] In fact, the Maximum Entropy Principle is most helpful in cases where repetitions are impossible or irrelevant (Jaynes 1962).

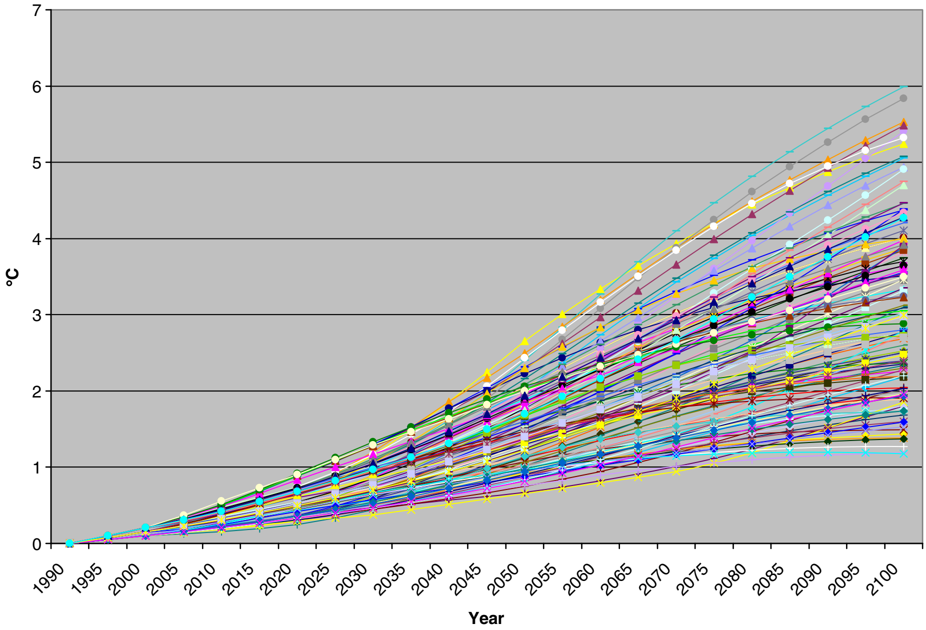

Fig. 1 Global mean temperature scenarios for 2100 using 38 SRES emission scenarios and three climate sensitivities

Figure 1, shows a possible representation of the range of climate change scenarios, consisting of 114 global mean temperature scenarios using 38 SRES emission scenarios (families A1, A2, B1 and B2) and three sensitivities.[7] In this case, the mean global temperature change for year 2100 ranges from 1.18°C to 5.99°C.

Table 1 summarizes three more possible representations of the state of knowledge and their corresponding entropy[8] and relative entropy. The relative entropy is calculated as the ratio of a particular state of knowledge and the widest range considered, in this case the range of the IPCC's likely ranges.

As is shown in Table 1, the selection of the range that would be used for representing the state of knowledge can result in trading uncertainty by ignorance. In this case, part of the range of possible values is excluded from the beginning of the analysis, limiting the information available for decision-making and biasing the resulting probability distribution and subsequent assessments (for example, of the potential impacts of climate change). The IPCC's AR4 "best estimates", if taken as the representation of the current state of knowledge, entail a reduction of more than 50% of the uncertainty; the range in the IPCC's TAR would entail a reduction of an 11% and; the scenarios in Fig. 1, a reduction of a 6%. Evidently, this is only useful for comparing two or more different possible state of knowledge. Furthermore, the IPCC's likely ranges *do not* even attempt to represent the true range of possible

[7]These climate change scenarios were obtained using the MAGICC 4.1 software. The sensitivities used are 1.5°C (low), 2.6°C (medium) and 4.5°C (high).

[8]This value is computed for a (continuous) uniform distribution as $ln(b - a)$ where b is the upper limit and a the lower.

Table 1 Different possible states of knowledge and their entropy and relative entropy

State of knowledge	Entropy	Relative entropy
IPCC AR4 best estimates (1.8°C to 4°C)	0.79	0.47
IPCC TAR (1.4°C to 5.8°C)	1.48	0.89
MAGICC (1.18°C to 6°C)	1.57	0.94
IPCC AR4 likely ranges (1.1°C to 6.4°C)	1.67	1

outcomes (Meehl et al. 2007). According to the IPCC, each of the likely ranges is supposed to contain only about 66% to 90% of the probability mass.

Even though there are global temperature scenarios which project increases of more than 8°C, the IPCC's likely ranges are of special interest for two main reasons: they were produced by the consensus of the Working Group I and, they represent the range of possible global temperature increases that the IPCC's decided to make public to decision-makers and the general public. For these reasons, we consider that, for this paper, it is the relevant state of knowledge to be used.

It is important to realize that the probability distributions obtained by the Maximum Entropy Principle are conditional on the chosen state of knowledge, but that for any given state of knowledge the Maximum Entropy estimate has the same properties as described above and is the least biased estimate possible on the information at hand.

3.2 Current recommendations for handling uncertainties in climate change scenarios

Uncertainty has to be preserved as much as possible in order not to bias the subsequent analysis, but it also has to be processed in such a way that it is useful for decision making. This has been an important issue in climate change science and has produced a variety of recommendations for dealing with uncertainty in climate change assessments. At the present time, decisions regarding 2100 climate are to be made with several centigrade degrees of uncertainty, similar to the difference in temperatures from glacial and interglacial periods. Although decision-making regarding the climate change threat should be constantly updated as we approach this date and further information is obtained, most of the actions that are capable of producing important reductions in GHG emissions that can lead to increases in global temperature of less than 2°C (for example) at a feasible cost, are to be implemented in the next few decades (Parry et al. 2008a, b; Meinhausen and Hare 2008, among others).

Even though several emission scenarios and models are now available, some of the earlier literature on the assessment of the potential impacts of climate change in human and ecological systems was developed selecting a single climate change scenario as input (a subjective selection of a single emission scenario, a model and a climate sensitivity). Evidently, this did not provide a good representation of the available state of knowledge at the time and arbitrarily dismissed all uncertainty, producing a degenerate probability distribution.[9] This approach could only provide

[9] As stated in Jaynes (1962) "It is unreasonable to assign zero probability to any situation unless our data really rules out that case"

illustrative examples of the potential consequences of climate change under very restrictive assumptions and therefore was likely to provide a very biased estimation of risk. Until less than a decade, recommendations[10] suggested including at least two (high and low) climate change scenarios in order to offer contrasting visions of how climate could evolve and of its expected impacts/benefits (Hulme et al. 2002). Recommendations have changed little since then, and now what is recommended is that climate scenarios should be: consistent with global projections, physical plausible, applicable to impact assessment, and representative of future regional changes (IPCC-TGICA, 2007). Although all these recommendations provide important information, they still are limited to arbitrarily chosen possible outcomes (and assigning zero probability to any other) and they do not offer any measure of how probable these outcomes are, and therefore this key element for decision making is still missing. In all cases, uncertainty is arbitrarily dismissed. In practice, the scientific community devoted to climate change impact assessment has not been able to fully integrate uncertainty and all rely on stringent *subjective* assumptions such as the ones referred above (UNFCCC 2008a, b).

New methodologies that are more suitable for assessing the potential impacts of climate change under uncertainty (including the use of Monte Carlo methods, for example Preston 2006; Nawaz and Adeloye 2006; Gay et al. 2006) are being developed and implemented. Nevertheless, when the uncertainty becomes large and, for example, the same probability is assigned to each climate scenario, impact scenarios become non-informative and tend to a uniform distribution: as contrasting views of the world get farther apart and are "mixed" in the same proportion, the inferences about the future get more and more confusing. Even when other distributions are applied to the uncertainty, none of them can be considered *objective* and there is no measure of how much information is assumed for each of them, and therefore how much the impact scenario depends on this assumption. On the contrary, carefully integrating subjective (expert) information can produce probabilistic climate change scenarios that do not dismiss uncertainty and that are consistent with different subjective assumptions (information or expert judgment) regarding an average mean temperature change.

In this paper, the maximum entropy distribution is proposed as the least biased probabilistic representation of future climate values consistent with the expert or decision-maker information. The objective is to produce relevant information for decision-making according to different agents' judgment. The fact that this probability distribution maximizes the uncertainty consistent with the partial information that is available is the prime reason that justifies its use for inference (Jaynes 1957).

In order to obtain the maximum entropy distribution, two inputs are needed: (1) the range of possible outcomes of climate change scenarios obtained using climate models. The selection of this range has also an unavoidable subjective component, because the true range of possible future values of climate variables is unknown. For example, one important source of uncertainty, the climate sensitivity, is conventionally assumed to lay between 1.5°C (low), 2.6°C (medium) and 4.5°C (high), with a best estimate of 3°C (IPCC 2001b). A great number of climate simulations have been conducted assuming climate sensitivity is within this range. Nonetheless, it has

[10]See for example http://www.grida.no/climate/ipcc/emission/150.htm, http://www.ukcip.org.uk/scenarios/ukcip02/documentation/documents/UKCIP02_App.pdf

been shown that climate sensitivity could be well beyond the high estimate (IPCC 2001b). The choosing of this arbitrary range of possible values should aim to provide a good representation of the known uncertainties and be physically plausible; (2) an arbitrarily chosen average mean change[11] for the climate variable of interest. It is important to notice that, once the average mean change is subjectively chosen and a range of plausible values is selected as a representation of the state of knowledge, the rest of the procedure is completely objective. That is, the problem of finding the unknown probabilities becomes a pure inverse problem (Golan et al. 1996) that can be solved using optimization techniques. This is a straightforward application of the dice problem shown in Jaynes (1957, 1962), but in this case the average value is not known. The original six-sided dice problem of Jaynes assumes that the mean value is given; what we are doing is creating different scenarios for different (subjective) estimations of this average value and conditional on the range of values chosen for representing the state of knowledge. In the dice problem, scenarios could be constructed: if the average value were 4.5, what would be the probability of having, say, a 2 in the next toss? What would this probability be if the average value was 3? and so on. In each case the best predictions consistent with the information available would still be those obtained using the Maximum Entropy Principle. Different people can have different information about this average value and, therefore, the maximum entropy distribution will be different for each of them.[12] Evidently, not all of them can be correct. Nevertheless, each person will be making the best predictions consistent with their information (right or wrong). For two people that have the same information, the maximum entropy distribution will be exactly the same. The Maximum Entropy principle can produce probabilistic scenarios that are maximally noncommittal with regard to the missing information and still consistent with the expert (subjective) information and the state of knowledge available.[13]

3.3 Suggestions for the selection of the average mean change

In this section two main suggestions for selecting the average mean change are presented. The first one is based on the decision-maker's cautionary/reckless attitude regarding the climate change threat and the second consist in a modification of

[11]In most cases, this average mean change will not correspond to the most probable change. The average value "tilts" the probability distribution in favor of larger or smaller values depending on the side the chosen average is with respect to the "center" value that can be obtained dividing the uncertainty range by 2. If the chosen average value is (smaller) larger than the uncertainty range divided by 2, then outcomes (smaller) larger than the chosen average will be more probable.

[12]The same applies to different ranges of plausible values or representations of the state of knowledge. Once these subjective values are defined, the maximum entropy probabilities, conditional on these subjective assumptions, can be obtained.

[13]It is important to notice that: (1) the goal is not to produce a "correct" probability distribution, but to offer different probabilistic climate change scenarios that are consistent with a subjectively (arbitrarily) chosen average mean temperature (or precipitation) change; (2) The range of uncertainty is preserved, no storyline or emission scenario is assumed to occur for sure and every possible outcome has a positive probability (no possibility is ignored); (3) Physical climate models and emission scenarios are used only to enumerate the possible outcomes needed for statistical inference; (4) The probabilities that are obtained are conditional on the subjective chosen mean value and the representation (range of values) of the state of knowledge.

one of the current recommendations for dealing with uncertainty in climate change scenarios.

3.3.1 Decision-maker's attitude

The decision-maker's attitude regarding the uncertainty surrounding climate change provides a useful way for selecting the average value needed for obtaining a maximum entropy distribution. In the absence of additional information, the maximum entropy principle leads to a uniform distribution. This distribution will be considered as *neutral* given that it does not include any subjective additional information. Assuming that there is no true, undisputed information, that can be used to objectively assign higher/lower values to the different outcomes, if a particular agent decides to assign higher (lower) probabilities to the most favorable outcomes and smaller (higher) to the least favorable than a neutral agent would, the resulting probability distribution could be associated to a reckless (cautious) attitude. All agents display different attitudes towards uncertainty.

In this paper we will use the term cautious agent for those agents that when facing an uncertain situation will assign a higher subjective probability of occurrence to the least favorable outcomes than a neutral agent would. The particular level of cautious/reckless attitude shown by a decision-maker would lead him to select the particular average mean change needed for constructing a probabilistic climate change scenario that best reflects his attitude towards the threat of climate change. That is, the average mean change chosen by a particular decision-maker will reveal his type of attitude towards uncertainty. Different decision-makers with different attitudes towards uncertainty will have different probabilistic climate change scenarios. A cautious decision-maker will tend to select a higher average mean change which implies higher probabilities for higher temperature increases, demonstrating his preoccupation of possibly underestimating the probabilities of these less favorable outcomes (higher impacts). Some actions consistent with this type of attitude could be, for example, devoting important international lobbing efforts for reducing GHG gases emissions, investing significant amounts of resources for preparing for the potential impacts of climate change and for adaptation.

On the other hand, a reckless decision-maker will tend to select a lower average mean change, assigning lower probability of occurrence to the least favorable outcomes, showing a lower level of concern for the possibility of underestimating the probabilities of occurrence of these outcomes. Nevertheless, for most of the range of reckless attitudes, this type of decision-maker still believes that large increases in temperature are possible, and therefore he will still devote some resources to mitigation, adaptation and for being prepared for the potential impacts of some of the most unfavorable outcomes.

A neutral decision-maker would choose an average mean change that assigns the same probability to each possible outcome; his attitude does not lead him to give higher or lower weights to any possible outcome. The noncommittal or "politically correct" decision of the authors of the Special Report on Emissions Scenarios (IPCC 2000) of not offering any judgment as to the preference for any of the emission scenarios and of clearly stating that no policy recommendation is intended could be interpreted as a neutral position. Figure 2 shows some possible maximum entropy distributions reflecting different types and degrees of these attitudes towards uncertainty.

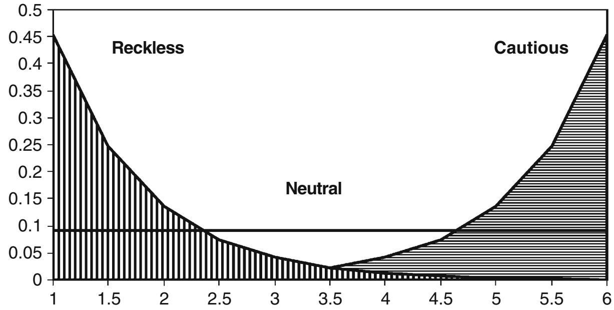

Fig. 2 Possible maximum entropy distributions for three types of agent: reckless, neutral and cautious

Assuming that decision-makers will assign the available resources for adaptation, mitigation and impact remediation and avoidance accordingly to their subjective probabilistic climate change scenarios, it can be seen that there are efficiency and cost/benefit relations implied. First, a neutral agent would distribute the resources available among all possible outcomes not showing a particular preference to avoid or prepare for any outcome. When resources are scarce this might no be the most efficient assignation. Selecting a lower average mean change in temperature could imply higher costs due to lower preparation (adverse impacts) for the least favorable outcomes, but reduces the possible costs due to over-preparation. A cautious agent would focus most of the available resources on the least favorable outcomes and, therefore, in case of one of these outcomes actually realizes, be better prepared. Nevertheless, he is risking in incurring in higher preparation costs than actually needed.

It can be useful to keep track of how much cautious/reckless a particular average mean change represents. For this purpose, a linear function whose domain is the values obtained from the collection of climate change scenarios and its range is -1 to 1 is proposed as a cautious/reckless index. The value of minus one represents the most reckless attitude while the plus one value is the most cautious. The linear function could be considered as having the disadvantage of assuming that this index increases monotonically for all possible values and therefore, it might not provide an adequate measure. Other non-linear functions (such as a cubic function) could provide a better way of measuring the changes in the index value that increases slowly for values close to the arithmetic mean of the range of uncertainty and that rapidly increase as the average mean change gets farther from this value (in this case 3.5°C). Nevertheless, it is recommended to restrain the cautious/reckless index to be an ordinal measure.

It is important to notice that trade between agents with different (even if slightly different) estimates of probabilities is possible. Therefore, a cautious agent could hedge trading risk with another less cautious agent.

3.3.2 Choosing arbitrary low and high average mean values

Another possible way for selecting the average mean change in climate change scenarios can consist on selecting arbitrary "low" and "high" average values. A linear function can also be utilized for keeping track of how much these values differ from the arithmetic mean value of the ensemble of climate change scenarios and therefore for providing a measure of how extreme our assumptions regarding the selected average mean change are (-1 corresponding to the lowest average mean value possible and 1 to the highest). Common recommendations such as selecting more than one climate change scenario can be easily adapted and, for example, "low" and "high" probabilistic climate change scenarios can be contrasted.

Measures such as the relative entropy and the information index can provide important information about how much the selected average mean change reduces uncertainty and of how informative this value is. These measures are helpful for keeping track of how much our probabilistic scenario depends on our arbitrary assumptions.

The relative entropy is calculated as the ratio of the entropy of a particular distribution and the entropy of a uniform distribution with the same support. This measure provides a quantitative measure of how much the maximum uncertainty (that is obtained using a uniform distribution) is reduced by imposing a different distribution for a given support. On the other hand, the information index[14] provides a measure of how much the particular probabilistic scenario depends on "expert" subjective judgment. A high information value corresponds to a small relative entropy value, implying that a large part of the original uncertainty has been removed. That is, the resulting distribution depends heavily on the subjective information added.

The reckless/cautious index, the relative entropy and the information index can provide a way of classifying the attitude of the different agents, of quantifying how much information they are assuming to have and, how much of the uncertainty is being removed.

4 Probabilistic climate change scenarios for 2100

The most important applications of this methodology for constructing probabilistic climate change scenarios could be in risk and potential impact assessment using Monte Carlo techniques, microeconomic theory (particularly, choice under uncertainty and game theory) and policy making. Nevertheless, in this section, for the sake of simplicity, a very simple example is given. In addition, the results of this example can be compared to the frequentist results shown in Schneider (2001). Different probabilistic climate change scenarios are developed, each of them consistent with the state of knowledge represented by the IPCC's likely ranges and different degrees of *reckless/cautious attitudes and arbitrary assumptions* about the average mean global temperature change for 2100. Then, the probabilities of exceeding some threshold values identified as "dangerous" climate change (Joachim et al. 2006; Bruckner and Schellnhuber 1999; Schneider and Lane 2006, for example) implied by different assumptions are explored. It is important to notice that these probabilities are the best predictions possible with the available information and a particular set of arbitrary assumptions chosen by the decision-maker.

Figure 3a shows the probabilities of exceeding "threshold" values of 2°C and 3.5°C increases in global temperature which have been identified as dangerous climate change. These probabilities are produced using different maximum entropy

[14]The information index can be calculated as one minus the relative entropy.

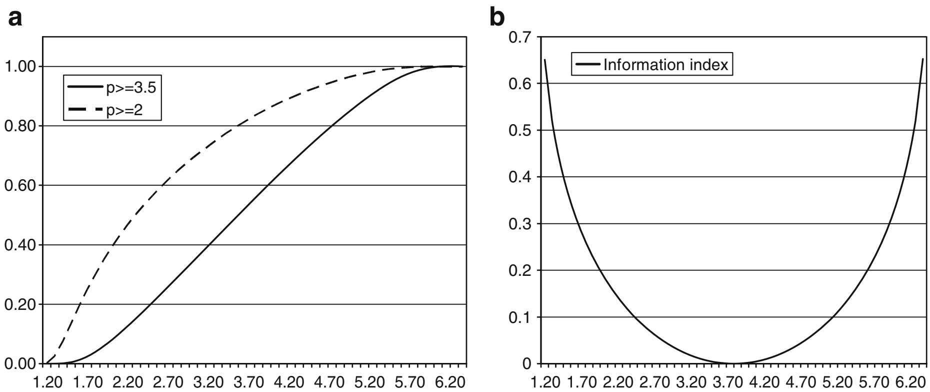

Fig. 3 **a** Probabilities of exceeding "threshold" values of 2°C and 3.5°C corresponding to different subjective selections of mean average change in the interval [1.2°C, 6.3°C]. **b** Information index corresponding to different subjective selections of mean average change in the interval [1.2°C, 6.3°C]

distributions for a range of arbitrary average mean change values from 1.2°C to 6.3°C. As can be seen, the probabilities of surpassing these threshold values hold non-linear relations with respect to the arbitrarily average mean change that is chosen, rapidly increasing for small values of the average mean change and slowly approaching to one for large values. Non-linear relations are also held with respect to different values of the proposed linear reckless/cautious index. Figure 3b presents the corresponding information index for the different average mean changes chosen. The information index can be interpreted as a measure of how much each of these probabilistic scenarios depend on the subjective information that is added for its construction. A non-committal, non-informative average mean change, corresponding to the arithmetic mean of the full range of outcomes (in this case 3.75), will produce a uniform distribution. In this case, no additional subjective information is added, and uncertainty is maximized. This is the case of a decision-maker that thinks that there is no reason to believe that a particular outcome is more or less likely to occur (Principle of Insufficient Reason of Bernoulli and Laplace). Any other probability assignment becomes increasingly dependent on the agent's subjective judgment as his assumptions about the average mean change get farther from the arithmetic mean of the set of possible outcomes.

Table 1 presents estimations for particular values of the arbitrary chosen average mean value and their corresponding degree of reckless/cautious attitude. As can be seen in this table, the probabilities of surpassing the chosen threshold values are quite large for most of the possible range of reckless/cautious attitudes (chosen average mean temperature changes). Only for agents showing very high levels of reckless attitude, the probabilities of surpassing 2°C are negligible. For example, the first row Table 1 shows that when choosing an average mean change of 1.2°C, an extremely reckless attitude is assumed (a value of −0.96 of the linear reckless/cautious index). In this case, as revealed by the information index, the decision-maker's subjective selection of the average mean change importantly reduces the relative entropy and therefore is considered to be highly informative and the resulting probability distribution is highly influenced by his subjective information. Evidently, a decision-maker showing this particular level of reckless attitude is assuming a great risk of large losses due to the potential impacts associated to higher temperature increases. Given this probability assignment, the decision-maker does not have important incentives for eliminating the worst possible outcomes through international lobbing for reducing GHG emissions, investing in cleaner technology, or for building adaptation capacities.

The probabilities of exceeding the 2°C and 3.5°C thresholds for a slightly less reckless agent, who chooses an average mean change of 2°C, increase rapidly up to 39.3% and 8.1% respectively (see Table 2, row 2).

On the other hand, an agent with a high level of caution (a 0.85 value of the linear reckless/cautious index) will produce another informative probability distribution that gives almost certain probability of occurrence of temperatures higher than 2°C and 3°C (see row 7 of Table 2). It is relevant to notice that, as indicated by the value of information index, these conclusions greatly depend on the agent's subjective judgment. This agent will act as if the worst outcomes are to occur and will try as much as he can to avoid them.

It is important to consider that this is a sequential process. Perception of risk for all agents will change as time goes by and a better picture of climate change is available. If climate change impacts continue to manifest, possible higher impacts will become more credible menaces and agents will update their perception of risk and beliefs about their occurrence.

If a neutral attitude is assumed, our subjective beliefs do not lead us to assign higher (lower) probabilities to any outcome, then the probabilities of surpassing 2°C and 3.5°C are quite large (83.3% and 55.6% respectively) and it can be questioned how many agents (governments) are acting as (at least) neutral regarding climate change scenarios. The limited success of international negotiations (such as in the case of the Kyoto protocol) might reveal that at present time a reckless attitude is the dominant way of thinking.

Schneider (2001) shows two different frequentist estimations of the probability of surpassing 3.5°C. Using 18 GCMs and six illustrative SRES scenarios he obtains a 23% probability for this event, while using the same GCMs but only the highest and lowest SRES illustrative scenarios he finds a probability of 39%. These results are comparable to our estimations for moderate-high levels of reckless attitudes.

Table 2 Probabilities of exceeding some threshold values identified as "dangerous" climate change

Average mean change	Linear reckless/ cautious index[a]	H(p)	$P(x \geq 3.5)$	$P(x \geq 2)$	Relative entropy	Information index	Lambda[b]
1.2	−0.96	1.392	0.000	0.002	0.349	0.651	6.93
2.0	−0.67	3.247	0.081	0.393	0.814	0.186	1.03
3.0	−0.30	3.870	0.346	0.694	0.970	0.029	0.324
3.75	0	3.989	0.556	0.833	1	0	0
4.0	0.11	3.976	0.623	0.870	0.997	0.003	−0.104
5.0	0.48	3.643	0.868	0.971	0.913	0.087	−0.598
6.0	0.85	2.501	0.999	1.00	0.627	0.373	−2.232

[a]This is also a linear measure of how far the selected average mean change is from 3.5°C

[b]Lambda is the Lagrangian Multiplier and its value is the rate of change in the objective function (entropy) as the constraint is relaxed

Selecting two arbitrarily "low" and "high" average mean changes can provide a particular decision-maker intervals for the occurrence of some event. For example, choosing a 2.5°C and a 3.5°C average mean changes produces a [56.7%, 79.2%] for surpassing a 2°C threshold, and a [20.7%, 48.6%] for 3.5°C.

5 Conclusions

This paper offers a methodology to process the objective and subjective information available in a way that is consistent with the original SRES intentions: "Preferences for the scenarios presented here vary among users... While the writing team as a whole has no preference for any of the scenarios, and has no judgment as to the probability or desirability of different scenarios, the open process and initial reactions to draft versions of this report show that individuals and interest groups do have such judgments".[15] The objective of this methodology is to produce relevant information for decision-making according to different agents' judgment and levels of reckless/cautious attitude. In this paper, the Maximum Entropy Principle is used for assigning probabilities to climate change scenarios when only partial information is available. These estimates have desirable properties such as: they are the least biased estimate possible on the information at hand; maximize the uncertainty (entropy) subject to the partial information that is given. That is, it produces the closest probability assignment to a uniform distribution consistent with what is known; the maximum entropy distribution assigns a positive probability to every event that is not excluded by the given information; no possibility is ignored; The probabilities obtained in this manner are the best predictions possible with the state of knowledge and subjective information that is available.

Now, in the wake of the release of the IPCC's Forth Assessment Report (AR4), it is fundamental to discuss the new methodologies that are being proposed for dealing with uncertainty and to bring forward some of our main concerns regarding this issue. Particular attention will be paid to the Summary for Policy-Makers (SMP-AR4) of Working Group I.

First of all, let's not forget that the SPM-AR4 information is intended for policy-makers, which, in general, have no formal training in climate and/or uncertainty. When presenting "official" and "objective" "best estimates" and "likely ranges", this will be the information that will be used for decision-making. These probabilities could be interpreted as "facts" such as "when flipping a fair coin, tails have a 50% chance of occurring". This is not the case. It is not possible to produce "objective" probabilities and, therefore, best estimates and likely ranges are nothing but a device and can be misleading. The IPCC is taking a strong position telling how decision makers should weight the possible climate change (temperature) scenarios and should be aware of the responsibility of reducing uncertainty in this manner, unjustifiable in our opinion. Here are some of our reasons for making this judgment of the way uncertainty is managed in the IPCC's AR4:

- As stated before in this paper, it is important to realize that a unique and "objective" judgment regarding the probabilities of the different climate change

[15] http://www.grida.no/climate/ipcc/emission/142.htm#anc1

scenarios is neither achievable nor desirable and therefore subjective information plays (and should play) a crucial role (whether we like it or not). The IPCC's "best estimates" and "likely ranges" are based on frequentist estimations that, as stated before, are really nothing but another way of setting subjective probabilities. Therefore, the IPCC's AR4 estimations should be regarded as subjective as any other estimation.

- As stated in Chapter 10 of the WGI of the IPCC's AR4, all ensembles used in the Report are "opportunity" ensembles, with no sampling methodology and are not expected to represent the full range of uncertainty. This characteristic makes the statistical interpretation of the range and statistical measures such as central tendency and dispersion measures even more problematic.
- The "best estimates" are based on the empirical fact that the mean of an ensemble *tends* to provide a better *forecast* than any of its individual members because if the different members are *independent and equally probable*, then individual biases tend to cancel. This is not the case in climate change scenarios. Two main issues arise: (1) There is no warranty that a particular model or the ensemble mean will continue to perform as well in the future as it does now. (2) One of the main characteristics of climate change scenarios is the dominance of epistemic uncertainty, due to the lack of knowledge (for example, climate sensitivity and cloud parameterizations, among others). Techniques used for forecasting weather (and short-term climate) are suitable for aleatory uncertainty. These are different types of uncertainty and epistemic uncertainty cannot be solved using techniques that were developed for addressing aleatory uncertainty.
- The "likely ranges" represent anywhere from 66% to 90% confidence intervals (about a plus/minus one or two standard deviation from the mean in a normal distribution). These criterion leaves out the scenarios that represent higher risk. It is worth remembering that in any risk assessment situation, a great interest is placed in low-probability/high-impact outcomes. For the common decision-maker, scenarios outside these likely ranges will not be considered and therefore uncertainty is dismissed by this subjective definition of "likely ranges".
- Likely ranges are constructed as a fixed proportion of the multi-model average: For any given average value, likely ranges or 66–90% probability intervals can be constructed by adding 60% of the multi-model mean value and subtracting 40%. This is an oversimplification and posses the question of why running such a "large" number of models.
- The IPCC keeps avoiding the emissions uncertainty and therefore, these probabilistic scenarios are conditional on the emission scenario for which we have no information regarding its probability of occurrence.
- According to the IPCC's AR4 these best estimates and likely ranges are included explicitly to avoid loss of "policy-relevant information". We strongly believe that this information is misleading and therefore should be used with caution.
- If we were to use the "best estimate" values for each emission scenario family as the average mean change needed to apply the methodology in this paper, each of these "best estimates" would imply a reckless attitude. As a product of the constant proportions used to construct the "likely ranges" (plus 60% minus 40% of the multi-model mean value), the resulting distribution would be biased towards the smaller increases in global temperature, assigning small probabilities to the worst outcomes.

Although uncertainty can be reduced improving our understanding and the ability of climate models to mimic the climate system (and also by eliminating "outlier" models if and only if are proven to be inadequate), it is important to bare in mind that one of the main sources of uncertainty comes from emission scenarios. That is, a considerable amount of the uncertainty can be only eliminated through policy: if actions are taken to reduce emissions the largest temperature changes could be avoided; on the other hand if no action is taken the most probable result is that the lower temperature changes will be eliminated. Any other way of reducing uncertainty is just arbitrary and increases our ignorance (Schneider 2003).

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